Confinement and chiral symmetry breaking of 4d QCD-like theories with small 2-torus compactification

Wednesday, August 25, 2021 10:08

Confinement & Chiral symmetry breaking of 4d QCD-like theories w/ small T^2

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Plan
- Introduction
  - Confinement of 4d gauge theories
    - monopole condensation scenario
    - center vortex scenario
  - Semi-classical confinement in low-dim.
- 2-torus compactification w/ 't Hooft flux
  - Anomaly
  - Unbroken 0-form center symmetry
  - Unbroken 1-form center symmetry
- Fermions, Chiral symmetry

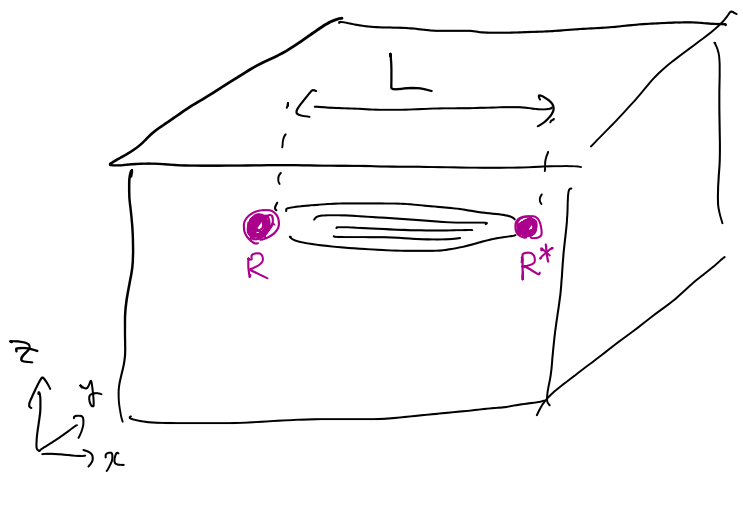
Introduction

4d Yang-Mills theory

$$S=\underset{\uparrow\text{ gauge coupling}}{\frac{1}{g^2}}\int \mathrm{tr}(F\wedge *F)+i\overset{\swarrow\text{ topological theta angle.}}{\frac{\theta}{8\pi^2}}\int \mathrm{tr}(F\wedge F).$$

$$F_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu+[a_\mu,a_\nu]\qquad SU(N)\text{ gauge field strength}$$

Confinement force between test quarks :

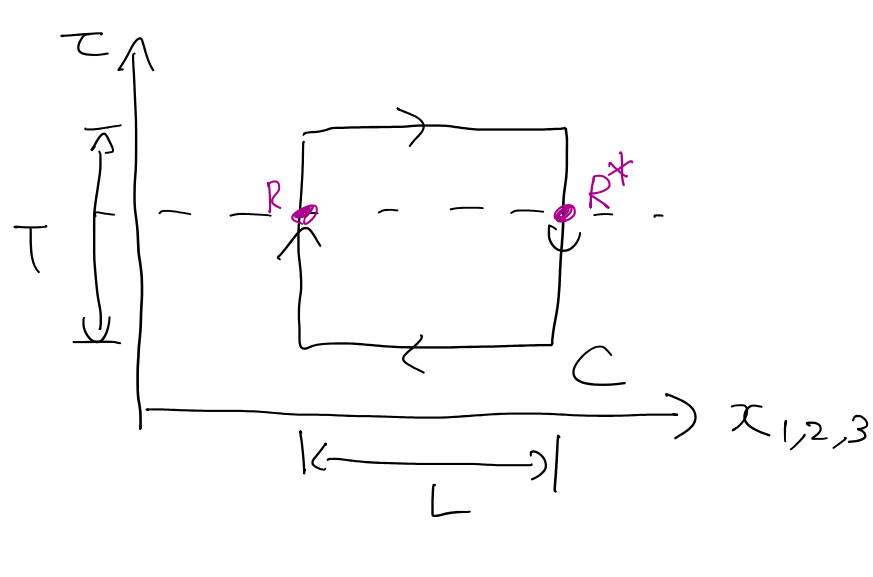


$$\frac{dV_R(L)}{dL}=\sigma_R.$$

$$(\text{i.e. }V_R(L)=\sigma_R\cdot L)$$

Wilson line operators $W_R(C)=\mathrm{tr}_R[\mathcal{P}\exp\oint_C a].$

$$\langle W_R(C)\rangle$$
$$\sim\exp(-T\cdot V_R(L))$$
$$\sim\exp(-\sigma_R\cdot \mathrm{Area}(C)).$$

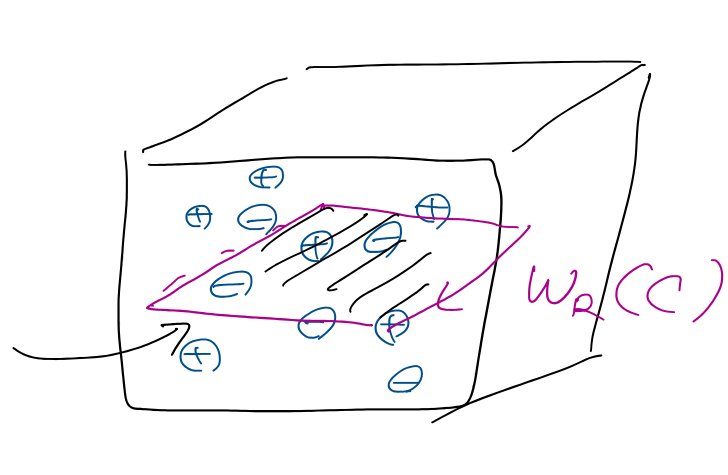


- scenarios of confinement mechanism
  - Monopole condensation
  - center vortex

Monopole condensation

ⓜ : magnetically charged particles.

Formation of confining string by dual Meissner effect

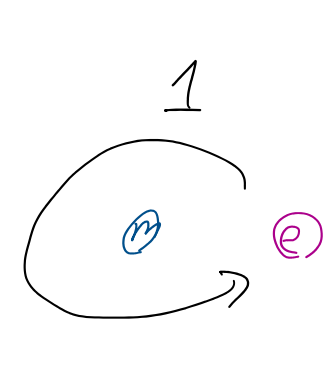


For $SU(N)$ gauge theory : Take $U(1)^{N-1}\subset SU(N)$

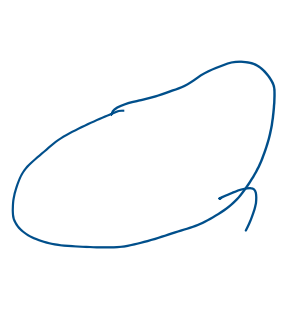
$$\begin{cases}\text{electric charge }\mu\in\Lambda_{\text{weight}}\simeq\mathbb{Z}^{N-1}\\ \text{magnetic charge }\alpha\in\Lambda_{\text{root}}.\simeq\mathbb{Z}^{N-1}\end{cases}$$

so that Dirac quantization is satisfied

$$e^{2\pi i(\mu,\alpha)}=1.$$

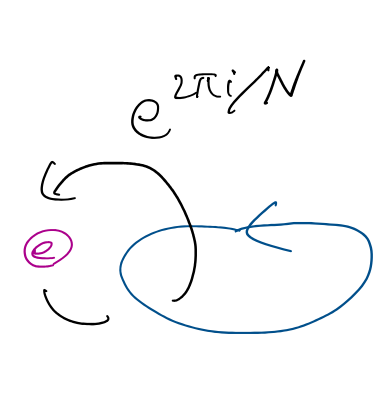


Center vortex

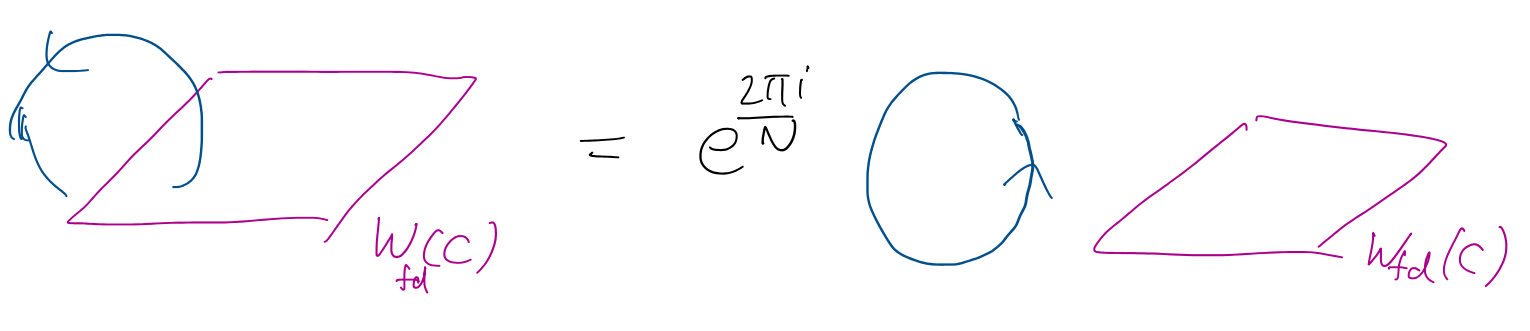


Center vortex is a magnetically charged string-type excitation.

Wilson loop & center vortex have a nontrivial mutual statistics. ⟹



Spacetime picture :



i.e. if center vortex pierces $W(C)$, it rotates the phase by $e^{\pm\frac{2\pi i}{N}}$.

This phase fluctuation makes $\langle W(C)\rangle$ very small if large center vortex is randomly excited. ⇝ Area law.

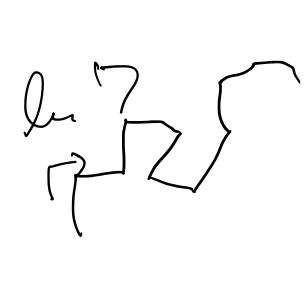
- Semi-classical description of confinement.

In 4D, both monopoles and center vortices are hard to give "semi-classical" explanation of confinement.

- Monopole is a loop in 4d spacetime
- Center vortex is a surface in 4d spacetime.

⇝ In order to have large loop/ surfaces,

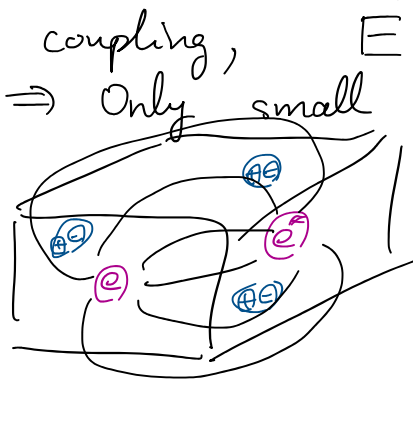
Energy density < Entropy factor.



$$\underset{\frac{1}{g^2}}{}\qquad \frac{1}{g^2}\int F_{\mu\nu}^2$$

In weak coupling, $E\sim O(\frac{1}{g^2})>S\sim O(1)$ ← typical factor for magnetic excitations

⇒ Only small loops/ surfaces can be created.



Charges are deconfined.

In 3D, the situation becomes better :

- Monopole is a point
- Vortex is a loop. ← Still, semi-classics is not available ↙ genuine 3D gauge theories

Semi-classical description of confinement by dilute monopole gas. (Polyakov, '77)

S^1 compactification of 4D YM + Adj. fermions becomes such 3D confining theory (Ünsal 2007- Davies, Hollowood, Khoze, 2000)

Idea In 2D, vortex is a point.
Can we get a new semi-classical confinement of 4D YM by compactification with T^2 ?

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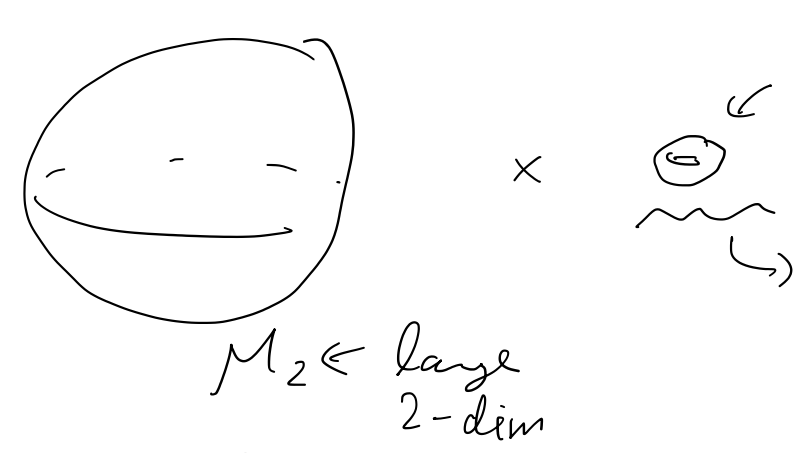
2-torus compactification

$M_4=$



M_2 ← large 2-dim

↖ 2d effective gauge theory.

- Naive T^2 compactification

$$\frac{1}{g^2}\int F\wedge F\Big|_{4d}\supset\frac{1}{g^2}\int (F_{34})^2dx^3dx^4$$

$$\boxed{F_{34}=0}\text{ classically}$$

Effective dynamics

$$P_3=\mathcal{P}e^{i\int a_3dx^3},\qquad P_4=\mathcal{P}e^{i\int a_4dx^4}.$$

$$\begin{cases}P_3,P_4\text{ are indep. of }x_3,x_4\\ [P_3,P_4]=0.\end{cases}\Rightarrow P_3,P_4\text{ are diagonalizable.}$$

1-loop effective action (G-P-Y effective potential)

$$V_{\text{eff}}(P_3,P_4)=-\sum_{n_3n_4\in\mathbb{Z}_{\geq0}}\#\,\mathrm{tr}(P_3^{n_3}P_4^{n_4})$$

Minima : $\underbrace{P_3,P_4=\mathbb{1}}$ $\mathbb{Z}_N^{[0]}$ center are broken.

$$(\Leftrightarrow\ a_3,a_4=0)$$

2d effective theory is pure 2d $SU(N)$ YM.

$$\rightsquigarrow\langle W_R(\underset{\uparrow\text{inside }M_2}{C})\rangle=\exp(-C_2(R)\mathrm{Area})$$

$\mathbb{Z}_N^{[1]}|_{2d}$ is unbroken.

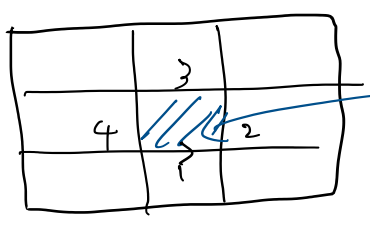
Summary for trivial T^2 : $\mathbb{Z}_N^{[1]}|_{4d}=\begin{cases}\mathbb{Z}_N^{[0]}\times\mathbb{Z}_N^{[0]} & P_3P_4\\ \mathbb{Z}_N^{[1]} & W_R\end{cases}$

- $\mathbb{Z}_N^{[0]}\times\mathbb{Z}_N^{[0]}$ is broken
- $\mathbb{Z}_N^{[1]}$ is unbroken
- 2d θ angle for pure $SU(N)$ YM is absent.

⟺ 4d θ has nontrivial dynamics of $SU(N)$ YM.

- T^2 compactification w/ 't Hooft flux.

$T^2\sim3\times3$ lattice.



$$\frac{1}{g^2}\sum_P\Big(1-\frac{1}{N}\mathrm{Re}\,\mathrm{Tr}(U_p)\Big)\qquad B_p\in\{0,\tfrac{2\pi}{N},\tfrac{2\pi}{N}2,\cdots,\tfrac{2\pi}{N}(N-1)\}$$

$$\xrightarrow{\text{'t Hooft flux}}\ \frac{1}{g^2}\Big(1-\frac{1}{N}\mathrm{Re}\big(e^{iB_p}\mathrm{Tr}(U_p)\big)\Big)$$

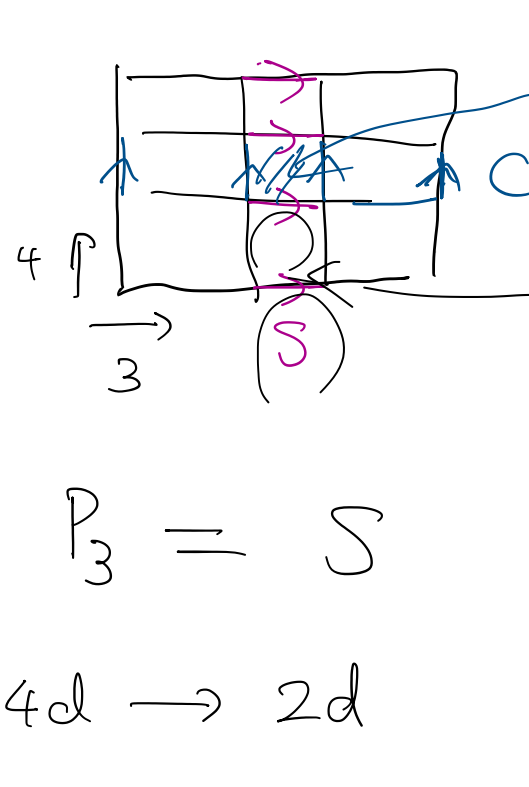
Maximize
$$\underbrace{\frac{1}{N}e^{iB_p}\mathrm{Tr}(U_p)}=1$$

$$\frac{1}{N}e^{\frac{2\pi i}{N}}\mathrm{tr}(U_1U_2U_3U_4)=1.$$

$$U_1=U_3^{-1}=S=\begin{pmatrix}0&1&&\\&\ddots&\ddots&\\&&\ddots&1\\1&&&0\end{pmatrix}$$

$$U_2=U_2^{-1}=C=\begin{pmatrix}1&&&\\&\omega&&\\&&\ddots&\\&&&\omega^{N-1}\end{pmatrix}\qquad\omega=e^{2\pi i/N}.$$

$$SC=e^{2\pi i/N}CS.$$


4
3
C
S
$\frac{1}{N}\mathrm{tr}(U_p)=e^{-2\pi i/N}$
$\frac{1}{N}\mathrm{tr}(U_p)=\frac{1}{N}\mathrm{tr}(S\cdot S^{-1})=1.$

$$P_3=S,\qquad P_4=C.:\ P_3P_4=\overset{\text{'t Hooft flux}}{e^{\frac{2\pi i}{N}}}P_4P_3$$

$$4d\to2d\qquad \mathbb{Z}_N^{[1]}|_{4d}=\mathbb{Z}_N^{[1]}|_{2d}\times\underbrace{\mathbb{Z}_N^{[0]}\times\mathbb{Z}_N^{[0]}}_{\text{unbroken}}$$

$$\begin{cases}\mathrm{tr}(P_3)=\mathrm{tr}(S)=0.\\ \mathrm{tr}(P_4)=\mathrm{tr}(C)=0.\end{cases}$$

$(\mathrm{tr}(P_3^{n_1}P_4^{m_1}P_3^{n_2}P_4^{m_2}\cdots)=0$ whenever it is trivial wrt $\mathbb{Z}_N^{[0]}\times\mathbb{Z}_N^{[0]})$

Unbroken $\mathbb{Z}_N^{[1]}$ becomes nontrivial

$$\mathcal{L}_{\text{eff},2d}\sim\frac{1}{g^2}\int F_{12}^2+\big|(\partial_1P_3+[a_1,P_3])\big|^2+\cdots$$

↑ Higgs $-a_1,a_2$ coupling.

⇒ All components of gauge fields become massive due to Higgsing.

⇝ Classically, by Higgs mechanism.

$$W_R(\underset{\uparrow\text{in 2d}}{C})=\underline{1}\ \leftarrow\text{perimeter law.}$$

$\mathbb{Z}_N^{[1]}$ seems to be broken ??

Lore : In QM, 0-form sym. is not broken
(If it is broken at classical level, instanton calc. ⟶ at quantum level, sym. is restored)
⇝ In 2d QFT, 1-form sym. is not broken.

2d U(1) gauge-Higgs model.

$$\mathcal{L}=\frac{1}{g^2}|\partial_1a_2-\partial_2a_1|^2+|(\partial_\mu+iNa_\mu)\phi|^2+\underset{\substack{\shortparallel\\0\ \ |\phi|=v.}}{V(|\phi|^2)}$$
$$+i\frac{\theta}{2\pi}da$$

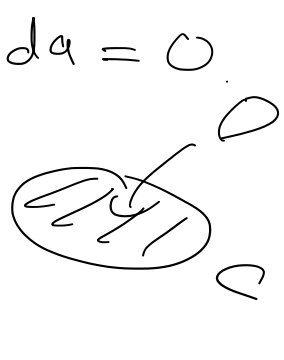
$\phi\sim ve^{i\varphi}$ at low energy.

$$\mathcal{L}_{\text{eff}}\sim\frac{1}{g^2}|f|^2+v^2\underbrace{|Na_\mu+\partial_\mu\varphi|^2}_{\Downarrow}+i\frac{\theta}{2\pi}da$$

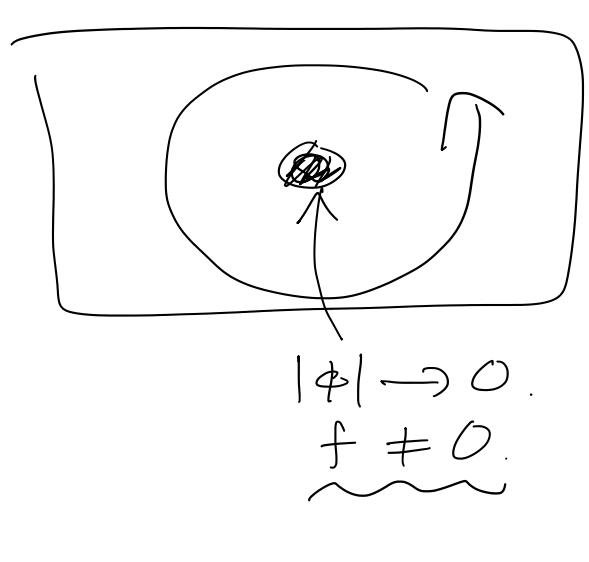
$$\underline{a=-\frac{1}{N}d\varphi},\qquad f=da=0$$

$$W(C)=e^{i\oint_C a}=e^{i\int_D f}=\underline{1}$$

perimeter law.



However, $\underline{\varphi\to\varphi+2\pi}$



$$\int_{\mathbb{R}^2}f=\int_{S^1_\infty}a$$
$$=-\frac{1}{N}\int d\varphi$$
$$=-\frac{2\pi}{N}$$

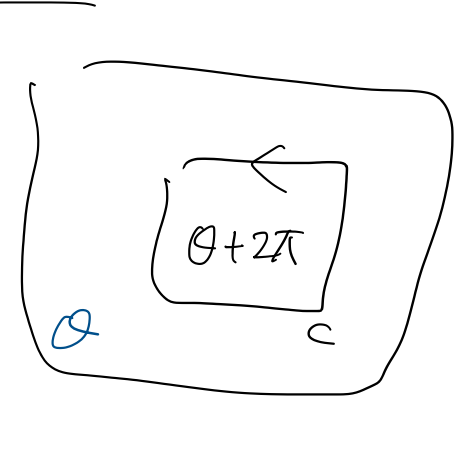
Vortex amplitude $\sim e^{-S_{\text{vortex}}}e^{-i\frac{\theta}{N}}$.

Dilute vortex gas approximation

$$\underline{E(\theta)\sim2e^{-S_V}(1-\cos(\tfrac{\theta}{N}))}.$$

$$\langle W(C)\rangle=\exp\Big(-\mathrm{Area}\big(\underbrace{E(\theta+2\pi)-E(\theta)}_{\neq0.}\big)\Big)$$

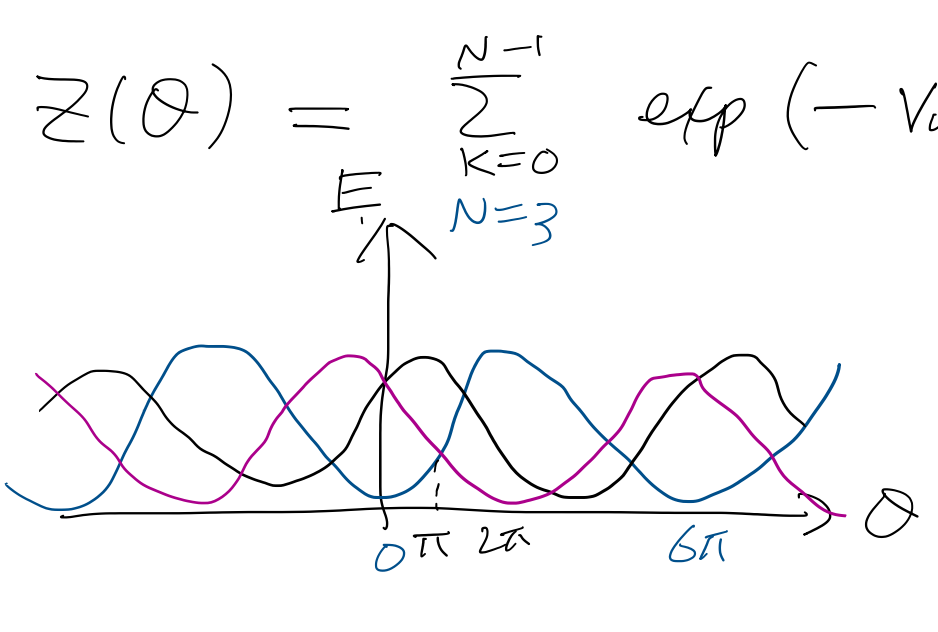
$$e^{i\oint_C a}=e^{i\frac{\boxed{(2\pi)}}{2\pi}\int_D da}$$



⟶ $\mathbb{Z}_N^{[1]}$ is restored due to vortex gas.

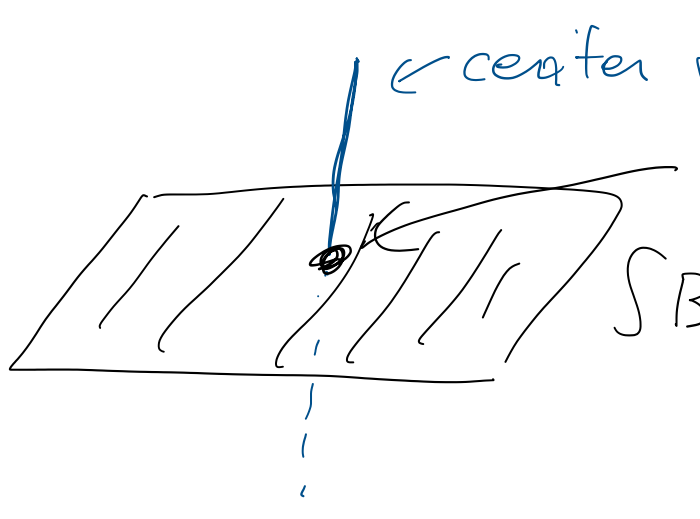
$$E(\theta+2\pi)\neq E(\theta),\qquad Z(\theta)=Z(\theta+2\pi)$$

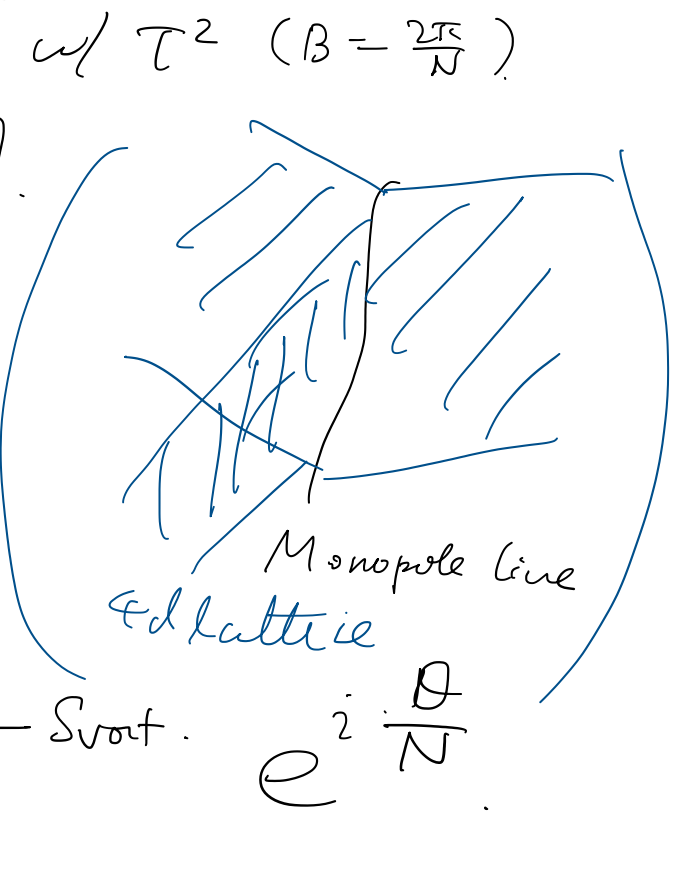
$$\rightsquigarrow Z(\theta)=\sum_{k=0}^{N-1}\exp(-\mathrm{Vol}\cdot E(\theta+2\pi k)).$$



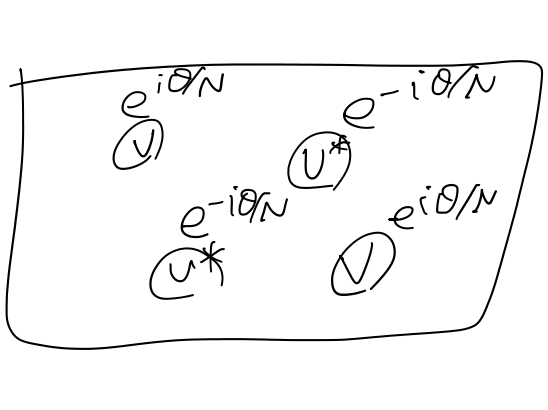
Claim : Similar thing occurs also for 4d YM w/ T^2 $(B=\frac{2\pi}{N})$

Center vortex restores $\mathbb{Z}_N^{[1]}$.





∴ Center vortex amplitude $\sim e^{-S_{\text{vort}}}e^{i\frac{\theta}{N}}$



Dilute gas ⟹ $E_{g.s.}(\theta)\sim e^{-S_V}(1-\cos(\frac{\theta}{N}))$

Comment Yamazaki - Yonekura

$$M_4=\underbrace{\overbrace{S^1\times S^1}^{\text{small}}}_{\int B=\frac{2\pi}{N}}\times\overbrace{S^1\times\mathbb{R}}^{\text{large}}$$

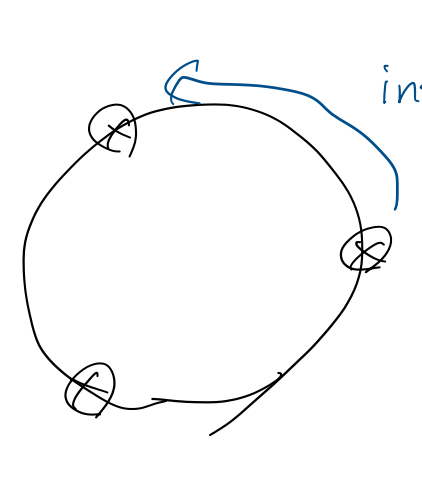
⟹ effective 2d theory on $\overset{\text{large}}{S^1}\times\mathbb{R}$

$=$ 2d $\underset{\substack{\uparrow\\ \text{deformed}}}{\mathbb{C}P^{N-1}}$ σ-model on $S^1\times\mathbb{R}$ w/ $\mathbb{Z}_N$-twisted compactification

so that $PSU(N)\supset\mathbb{Z}_N\times\mathbb{Z}_N$

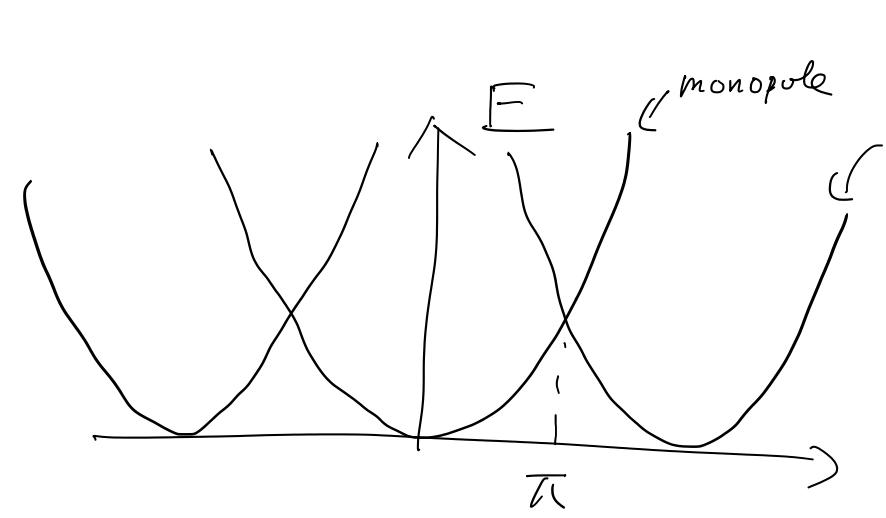
Classically. P along large S^1
$=1,e^{2\pi i/N},\cdots,e^{2\pi i/N(N-1)}$ N classical vacua

instanton jump $\underset{\sim\sim}{e^{i\theta/N}}$



Anomaly

4d YM



$$Z_{\text{mon}}[B]=1$$
$$Z_{\text{dyon}}[B]=e^{i\frac{N}{8\pi^2}\int B^2}$$

2π-periodicity of θ & $\mathbb{Z}_N^{[1]}$ is 't Hooft anomalous.

$$Z_{\theta+2\pi}[B]=\underbrace{e^{i\frac{N}{8\pi^2}\int B^2}}_{\text{anomalous phase of }\theta\text{ periodicity w/ 't Hooft twist }B}Z_\theta[B]$$

This anomaly still survives after 2d compactification / 't Hooft flux.

- Naive compactification $\underset{1,2}{M_2}\times\underset{3,4\text{ directions}}{T^2}$

$$B=\frac{1}{2}B_{\mu\nu}dx^\mu\wedge dx^\nu$$
$$=\underbrace{B_{12}dx^1\wedge dx^2}_{\mathbb{Z}_N^{[1]}|_{2d}}+\underbrace{B_{13}dx^1\wedge \textcircled{dx^3}}_{\mathbb{Z}_N^{[0]}\text{ gauge field}\wedge PD[S^1_3]}+\cdots$$

$$\int_{M_2\times T^2}B\wedge B=\int B_{12}\underbrace{\cancel{B_{34}}}_{\substack{\shortparallel\\0}}+\underbrace{B_{13}}_{A_3}\underbrace{B_{24}}_{\textcircled{A_4}}\quad \mathbb{Z}_N^{[0]}|\ P_4\to e^{\frac{2\pi i}{N}}P_4.$$

$$2d:\ \overset{B}{\mathbb{Z}_N^{[1]}}\times\overset{A_3}{\mathbb{Z}_N^{[0]}}\times\overset{A_4}{\mathbb{Z}_N^{[0]}}$$

$$Z_{\theta+2\pi}[B,A_3,A_4]=\underbrace{e^{i\frac{N}{2\pi}\int A_3\wedge A_4}}_{\text{'t Hooft anomaly.}}Z_\theta[B,A_3,A_4].$$

4d θ periodicity anomaly ⇝ $E_{\text{conf}}(\theta+2\pi)\neq E_{\text{conf}}(\theta)$

2d trivial compactification

Anomaly matching
- θ periodicity is violated
- $\underline{\mathbb{Z}_N^{[0]}\text{ is spontaneously broken}}$ ↖ this is sufficient for matching anomaly

- 2d comp. / $\int B_{34}=\frac{2\pi}{N}$.

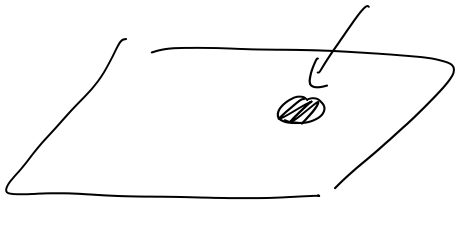
$$\int B\wedge B=\int B_{12}\underbrace{B_{34}}_{\frac{2\pi}{N}}+\int A_3\wedge A_4+\cdots$$

$$Z_{\theta+2\pi}[B,A_3,A_4]=\underbrace{e^{i\int B+iN\int A_3\wedge A_4}}_{\text{anomaly.}}Z_\theta[B,A_3,A_4]$$

Anomaly matching
- θ. periodicity is violated ↙ $\mathbb{Z}_N^{[0]}$ is unbroken ($\mathbb{Z}_N^{[1]}$ is restored by center vortex)
- Both $\mathbb{Z}_N^{[1]}$ & $\mathbb{Z}_N^{[0]}$ are broken.

Fermions

$e^{i\theta/N}$, $Q_{\text{top}}=\frac{1}{N}$.



$$\mathrm{index}(\not{D})=Q_{\text{top}}\sim\frac{1}{N}.$$

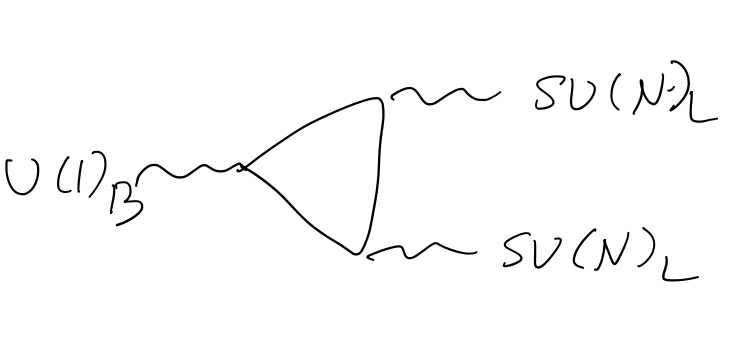
- Adj fermion λ : $\mathrm{index}(\not{D}_{\text{Adj}})=2N\,Q_{\text{top}}=2.$
$e^{-S_{\text{vort}}}\ \lambda\lambda.\quad\rightsquigarrow\ \langle\lambda\lambda\rangle\neq0.$

- Fd fermion ψ : $U(1)_B$ symmetry $=U(1)_q/\textcircled{\mathbb{Z}_N}$

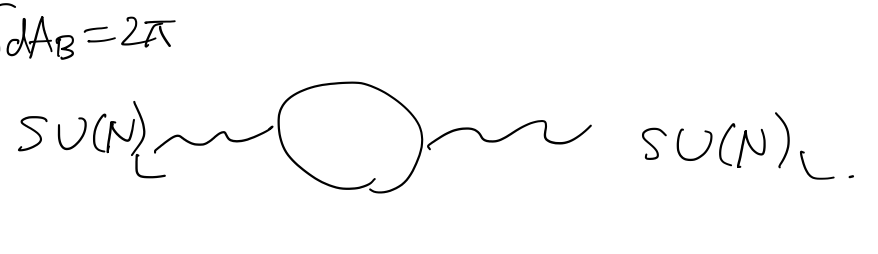
$$\underline{\int dA_B=2\pi}\quad\swarrow\text{baryon \# gauge field}$$

$$\Longleftrightarrow\int dA_q=\textcircled{\frac{2\pi}{N}}\quad\swarrow\text{quark \# gauge field}$$

↖ cancel. $SC=e^{\frac{2\pi i}{N}}CS.$



2d comp / $\int dA_B=2\pi$ ⟹



⟹ Low-energy EFT ∼ $SU(N)$ WZW.